Distributed Finite-Element Modeling and Control Approach for Large Flexible Structures

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This paper describes a new framework for the design of decentralized controllers for large flexible structures. In contrast to the existing decentralized control approach, in which the design begins with a given dynamic model of the flexible structure, the finite-element modeling and control design phases are integrated in this framework. Moreover, the integrated modeling and control design task are distributed among the individual structural components from which the large flexible structure is constructed. Controlled component synthesis is a method introduced for the distribution of integrated modeling and control. Using this method, controlled components are built and assembled into a controlled flexible structure that meets performance specifications. A simple truss structural control problem is employed to illustrate the design procedures and to demonstrate the potentials of the developed method for controlling very large dimensional truss structures.

Introduction

STRUCTURAL control has been an active area of research and development for the last decade, in response to an increasing demand in large space structure pointing and control requirements. Although there have been many innovations in large space structure (LSS) control technology, one of the more difficult hurdles is the communication gap between structural engineers and control engineers. The typical structural control problem begins with the structural engineer's finite-element modeling and model reduction efforts, from which a dynamic model of the structure is generated. The modeling phase is followed by a control design phase in which the structure model is further reduced to a manageable dimension for control design. Although this level of interaction has proven to be sufficient in the past for structures with a few dominant flexible modes in the control system bandwidth, it becomes inadequate when the complexity of the structural control problem increases as the control system bandwidth increases, the number of flexible modes inside this bandwidth increases as the structural material becomes lighter, and the physical size of the structure, as well as the number of actuators and sensors, grow by an order of magnitude. In the last decade, the need emerged for a distinctive hybrid engineering discipline, which had its origins in structural engineering and control system engineering. This new interdisciplinary field was designated control structure interaction (CSI) technology. The first technical conference solely dedicated to CSI was sponsored by NASA and the Department of Defense, and held in November 1986.

One of the key objectives in CSI technology is to develop methods and approaches in structural modeling and control while eliminating the technical inconsistencies that are hidden in structural and control system engineering practices. Such development, perhaps, is most needed in the design of distributed control for large space structures. Distributing or decentralizing LSS structural controllers means control action is localized. Since the structure models are developed without any consideration of the controller's action, the opportunity to model the local structural dynamic behavior in order to benefit the design of the distributed control laws is lost.

In this paper, a connection between structural dynamics and large-scale systems control is established to benefit the design of distributed structural controllers for LSS. The method developed herein detours from the conventional control system design path that begins with a model of the open-loop plant. Instead, the controlled plant is assembled from *controlled components,* in which the modeling phase and the control design phase are integrated at the component level. The developed method is labeled a controlled component synthesis (CCS) method to reflect that it is motivated by the well-developed component mode synthesis (CMS) methods, which are a collection of structural analysis methods that have been demonstrated to be effective for solving large complex structural analysis problems for almost three decades. The design philosophy behind CCS is closely related to that of the subsystem decomposition approach in decentralized control. The ideas behind CCS are stimulated by the subsystem decomposition viewpoint in large-scale system theory, and by the component mode synthesis methods in structural analysis. Connections between CCS and existing large-scale system decomposition techniques are established herein to build a control theoretic foundation for the developed method.

Subsystem Decomposition Issues

Decentralized control is intuitively appealing for structural control of large flexible structures. It offers simplified control system implementations that only require the feedback of local measurements to close the control loop for systems that may have a few hundred to thousands of control loops. Many decentralized control methods have been developed in the last decade, typically using one of these two approaches. The first is an *information constraint approach,* in which the control design uses the model of the large structure and seeks a feedback control solution to the problem, subject to constraints on the flow of the measurement information. The control action decentralization is inherent in any feasible solution of the problem satisfying the imposed constraints. In this approach, the design uses a model of the large structure in an optimization problem of the order of the model dimension. The theoretical foundation of this approach has been established by Davison and his co-workers in the 1970s. Decentralized control designs for large space structures using this approach can be found in the paper by West-Vukovich and Davison.[1] Because of the need to exercise a model of the large structure in the design, the designer is forced to reduce the model order to cope with computational resource limitations, at the expense of causing spillovers in the controlled structure.

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The second is a *subsystem decomposition approach,* in which the large structure model is first decomposed or partitioned into an interconnection of subsystem models. Local control designs are produced by designing controllers for the local decoupled subsystem models, which are derived from the large structure model by discarding the interconnections. These local designs are solved independently, thereby reducing the computational resource requirements for performing control designs for very large structures. The behavior of the locally controlled structure is analyzed using aggregation analysis methods. An aggregate variable is defined to represent the subsystem's dynamic interactions with the other subsystems. The order of the aggregate analysis problem is equal to the number of subsystems that are determined by the decomposition scheme. This approach is developed by Šiljak and his co-workers.[2] The work of Young[3,4] and Kida[5,6] adopt this approach for decentralized control designs.

Given a model of a large structure, a control system designer is faced with the problem of decomposing the given model into an interconnection of subsystem models, if a subsystem decomposition approach is to be adopted for decentralized control design. There are two ways to obtain the subsystem models. The first method converts the given structure model from its second order structure (SOS) model form to a state space (SSP) model, i.e., from the model

$$M\ddot{x}^n + Kx^n = f \tag{1}$$

where M and K denote the mass and stiffness matrix of the structure, x^n is a vector of displacements at the nodes of a finite-element model, and f is a vector of generalized forces applied at the nodes to an SSP model

$$\dot{x} = Ax + Bf \tag{2}$$

where

$$x = \begin{bmatrix} x^n \\ \dot{x}^n \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \tag{3}$$

An interconnected subsystem (ISS) model is obtained by decomposing the A and B matrices and the state vector x of the SSP model, which shows the interconnections of s subsystems:

$$\text{ISS}^i\text{:} \quad \dot{x}^i = A^{ii}x^i + B^{ii}f^i + \sum_{j\neq i} A^{ij}x^j + \sum_{k\neq i} B^{ik}f^k \qquad i = 1,\ldots, s \tag{4}$$

The local decoupled subsystem models are readily obtained from Eq. (4) by discarding the summation terms:

$$\text{SSP}^i_D\text{:} \quad \dot{x}^i = A^{ii}x^i + B^{ii}f^i, \qquad i = 1,\ldots, s \tag{5}$$

One of the attractive features of the subsystem decomposition approach is the use of local subsystem models, rather than the model of the structure, in designing decentralized controllers. However, from the foregoing discussions, it is clear that the large structure model must also be known to the designer in this approach. Thus, the use of local models in the local control designs is an option exercised by the designer to reduce the dimensionality of the associated computational problems and to improve the robustness of the controlled structure with respect to parametric perturbations. A more radical departure from the conventional large-scale system design path calls for the development of the decoupled local subsystem models without the use of the model of the structure. Controlled component synthesis adopts this new viewpoint on subsystem decomposition. Such a departure from large-scale system theoretic perspectives effectively coincides with the component mode synthesis viewpoints held by structural analysts for quite some time.

Distributed Finite-Element Modeling

The concept behind CMS was first introduced by Hurty.[7] (CMS is also referred to as substructure coupling. The term *component* instead of *substructure* will be used throughout this paper.) Craig[8] provided a survey of CMS methods developed between 1960 and 1976. The fundamental idea behind CMS methods is that a dynamic model of a large complex structure is to be built from models of its components. The components are first modeled as individual structures. These component model data are then processed to form a model of the large structure. The component models are typically developed using a finite-element method.

A critical design parameter in CMS methods is the choice of appropriate models or modes for the components. The selection of component modes directly affects the accuracy to which the model of the coupled structure, which is synthesized from component model data, approximates the finite-element model of the coupled structure. The development of CMS methods in the past two decades has been focused on the specification and the choice of component modes and on methods to handle the constraint equations. The recent work of Flashner,[9] in which a computational procedure for CMS using singular value decomposition is derived, offered an updated view of CMS. More details on CMS can be found in Meirovitch[10] and Craig.[11]

Two of the many approaches for choosing the component model data that will be considered in CCS are particularly noted here. Benfield and Hruda[12] introduced component modes, using boundary stiffness and inertia (mass) loading, in which the reduced mass and stiffness matrices of a component are reduced and "loaded" on the boundary coordinates of the adjacent component using the boundary compatibility condition. Craig and Bampton[13] proposed the use of constraint modes, in which the boundary coordinates are retained as component mode coordinates.

Controlled Component Synthesis

CCS is a framework for an integrated, component oriented, finite-element modeling and structural control design. Similar to CMS methods, a CCS method is developed on the premise that a large complex controlled structure is to be built from controlled components. The finite-element modeling and control design are performed for the individual components; the model of the large complex structure is assembled from the controlled components only for the purpose of performance evaluation and sensitivity studies.

The CMS methods would be a logical first choice for generating component models. However, if the component models developed by CMS are used in the design of the controller for the component, a subsequent issue remains: What model of the coupled structure should be used to assess the controlled structure performance? Although the CMS methods produce a synthesized model of the coupled structure, the accuracy and suitability of such models for closed-loop evaluations require further examination. On the other hand, the methodologies developed in the subsystem decomposition approach can be used to analyze the controlled coupled structure behavior, provided that models of the coupled structure can be developed, such that the approximations due to the assembly of the component models are suitably characterized for these methods. In the development of CCS methods, we elect to evaluate the system performance of the controlled coupled structure with systems theoretic tools developed in the subsystem decomposition approach. A high-fidelity model of the coupled structure, expressible in the form of an interconnected second order structure (ISOS) model, i.e.,

$$\text{ISOS}^i\text{:} \quad M^{ii}\ddot{x}^{ni} + K^{ii}x^{ni} + \sum_{j\neq i} M^{ij}\ddot{x}^{nj} + \sum_{j\neq i} K^{ij}x^{nj} = f^i \qquad i = 1,\ldots, s \tag{6}$$

will be adopted as an initial candidate for the coupled structure model, to be used in performance evaluation of the controlled coupled structure.

The CCS method developed herein adopts the following modeling and control design considerations at the component level. Instead of using either the boundary loading or the constraint modes approach, as outlined previously, we introduce a new approach for the development of component models based on boundary loading. For the design of controllers for the component, an interlocking control concept is developed to minimize the motion of the nodes that are adjacent to the boundary, thereby suppressing the transmission of mechanical disturbance from component to component in the coupled structure. A new synthesis procedure for deriving the coupled structure models from component models is also developed to provide better characterizations of the approximations due to the assembly of component models, with respect to an ISOS model.

Component Modeling for CCS

In Young,[14] we have illustrated some of the difficulties associated with characterizing the approximations in CMS, with respect to the high-fidelity model. Presumably, it is possible to dig deeper and develop the proper metric to measure the modeling errors due to CMS with component modal projection. However, since the ultimate goal is structural control, it is also possible to defer modal projection until after the control system design has been completed, and perform the so-called closed-loop model reduction. In the development of component models for CCS, we have chosen to follow the latter approach. Henceforth, we decide not to adhere to CMS and attempt to define high-fidelity component models whose approximations can be easily characterized.

A two-component structure, as shown in Fig. 1, will be used to outline the modeling and design procedure of the CCS method. Each of the structure components is composed of three finite elements, identified in the figure by Roman numerals; the element node points are identified by solid circles.

In CMS methods, the nodal coordinates of a component are partitioned into a set of internal (interior) coordinates, x_{is}, and a set of boundary coordinates, x_{bs}. In the CCS method, the internal coordinates are further subdivided into a group of internal boundary coordinates x_{ibs} and a group of internal coordinates x_{is}. Figure 1 indicates how the three groups of coordinates are defined. The boundary coordinates are coordinates of the boundary element, such as element III of component 1, which are on the boundary. The remaining coordinates of the boundary element are designated the internal boundary coordinates. (Herein, we use IIB coordinates to identify coordinates that are not on the boundary.) The remaining coordinates of the component are the internal coordinates.

Partitioning with respect to the three groups of coordinates, the finite-element model for a component has the form

$$\begin{bmatrix} M_{ii}^s & M_{iib}^s & 0 \\ M_{bii}^s & M_{ib}^s & M_{ibb}^s \\ 0 & M_{bib}^s & M_{bb}^s \end{bmatrix} \begin{bmatrix} \ddot{x}_{is} \\ \ddot{x}_{ibs} \\ \ddot{x}_{bs} \end{bmatrix} + \begin{bmatrix} K_{ii}^s & K_{iib}^s & 0 \\ K_{bii}^s & K_{ib}^s & K_{ibb}^s \\ 0 & K_{bib}^s & K_{bb}^s \end{bmatrix} \begin{bmatrix} x_{is} \\ x_{ibs} \\ x_{bs} \end{bmatrix} = \begin{bmatrix} f_i^s \\ f_{ib}^s \\ f_b^s \end{bmatrix} \tag{7}$$

The block bidiagonality of the mass and stiffness matrices is a direct consequence of the chosen finite-element mesh and the deliberate grouping of the coordinates for the component.

In developing a new approach to generate component models for CCS, we adopt the idea of boundary loading introduced by Benfield and Hruda,[12] yet decide not to slave the IIB coordinates to the boundary coordinates. In boundary loading, the stiffness matrix K^s of a component is reduced by eliminating the internal coordinates x_{is}. Let the component coordinates and stiffness matrix be partitioned

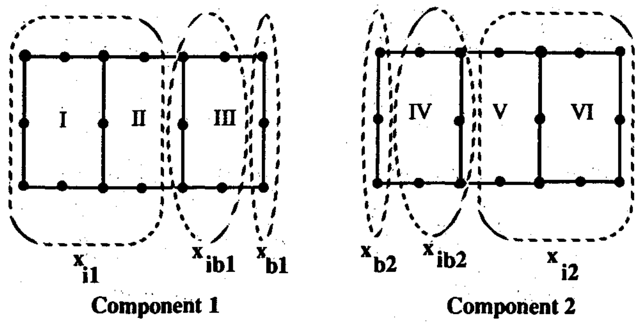

Fig. 1 A two-component structure modeled with finite elements.

$$x^s = \begin{bmatrix} x_{is} \\ x_{bs} \end{bmatrix}, \qquad K^s = \begin{bmatrix} K_{ii}^s & K_{ib}^s \\ K_{bi}^s & K_{bb}^s \end{bmatrix} \tag{8}$$

The reduced stiffness matrix with respect to the boundary coordinates is given by

$$K_b^s = T_b^{sT} K^s T_b^s \tag{9}$$

where

$$T_b^s = \begin{bmatrix} -K_{ii}^{s-1} K_{ib}^s \\ I \end{bmatrix} \tag{10}$$

The reduced mass matrix with respect to the boundary is derived following a similar process:

$$M_b^s = T_b^{sT} M^s T_b^s \tag{11}$$

In this new approach, instead of using the reduction matrix T_b^s, as given by Eq. (10), a truncation matrix, which eliminates both the internal and internal boundary degree of freedom, is used:

$$T_{bt}^s = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \tag{12}$$

The modifications to the mass and stiffness matrices of the component finite-element model are computed using Eqs. (9) and (11) in the boundary loading approach, but T_{bt}^s replaces T_b^s in these equations. The resulting modified mass and stiffness matrices for loading from the jth to the sth component are

$$M_{bb,iL}^s = M_{bb}^s + M_{bb}^j \tag{13}$$

$$K_{bb,iL}^s = K_{bb}^s + K_{bb}^j \tag{14}$$

Note that these modifications are restricted to the submatrices corresponding to the boundary coordinates, i.e., only the submatrices M_{bb}^s and K_{bb}^s are modified; the other submatrices in Eq. (7) remain unchanged. We call this new approach to component modeling, "isolated boundary loading." The modified mass and stiffness matrices due to this approach can be obtained alternatively from the finite-element modeling of an expanded component; that is, the original boundary of the component is extended one finite element into the adjacent component. The nodes of the expanded component consist of the original nodes of the component and the internal boundary coordinates of the adjacent component. This alternative derivation of the component model, which is physically explicit, results from the interpretation that using the transformation matrix T_{bt}^s is equivalent to the removal of the internal, and internal boundary degree of freedom, i.e., $x_{is} = 0$ and

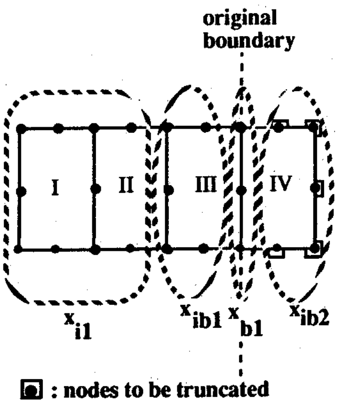

Fig. 2 Expanded component 1.

$x_{ibs} = 0$. For isolated boundary loading from the jth to the sth component, the sth component is expanded, as shown in Fig. 2. The modified mass and stiffness matrices are obtained from the mass and stiffness matrices of the expanded component by deleting the rows and columns corresponding to the nodes in the expanded portion.

Connections to Overlapping Decompositions

In addition to developing isolated boundary loading, we shall develop an alternative process, which is motivated by the following property of the modified matrices, for synthesizing a coupled structure model from the component models. The modified mass and stiffness submatrices are identical, irrespective of whether they are obtained by loading from the sth to the jth component, of from the jth to the sth component, i.e.,

$$M^s_{bb,iL} = M^j_{bb,iL} \tag{15}$$

$$K^s_{bb,iL} = K^j_{bb,iL} \tag{16}$$

Referring to the two component structure, the coupled structure model is synthesized in two steps. First, recombine the boundary coordinate equation in the model for the sth component

$$M^s_{bb,iL}\ddot{x}_{bs} + M^s_{bib}\ddot{x}_{ibs} + K^s_{bb,iL}x_{bs} + K^s_{bib}x_{ibs} = f^s_b \tag{17}$$

and that for the jth component

$$M^j_{bb,iL}\ddot{x}_{bj} + M^j_{bib}\ddot{x}_{ibj} + K^j_{bb,iL}x_{bj} + K^j_{bib}x_{ibj} = f^j_b \tag{18}$$

into a single equation for the boundary coordinates, in which x_{bsj} denotes the coordinates on the boundary.

$$M^s_{bb,iL}\ddot{x}_{bsj} + M^s_{bib}\ddot{x}_{ibs} + M^j_{bib}\ddot{x}_{ibj} + K^j_{bib}x_{ibj} + K^s_{bb,iL}x_{bsj} + K^s_{bib}x_{ibs} = f^s_b + f^j_b \tag{19}$$

In executing this step, we make use of superposition. Let

$$g^s_b \doteq M^s_{bb,iL}\ddot{x}_{bs} + K^s_{bb,iL}x_{bs} \tag{20}$$

$$g^j_b \doteq M^j_{bb,iL}\ddot{x}_{bj} + K^j_{bb,iL}x_{bj} \tag{21}$$

denote the forces due to the boundary motion of the sth component and the jth component, respectively. The hypothesis that the forces due to the boundary motion x_{bsj} and the boundary motion of the coupled structure result from the superposition of boundary forces and the motion of the components, respectively, i.e.,

$$g^{sj}_b = g^s_b + g^j_b \tag{22}$$

$$x_{bsj} = x_{bs} + x_{bj} \tag{23}$$

and the boundary matrix identities [Eqs. (15) and (16)], yield the final boundary coordinate equation. Note that we have used superposition instead of the boundary compatibility condition to synthesize the equations for the boundary coordinates.

The second step in the process involves a straightforward copy of the remaining submatrices from Eq. (7), arranging them correspondingly to produce a synthesized model that is identical to the finite-element model of the coupled structure, derived using standard finite-element methods.[15,16] The assembled finite-element model, given by the following equation, is referred to as a high-fidelity model of the coupled structure

$$M^{sj}\ddot{x}^{sj} + K^{sj}x^{sj} = f^{sj} \tag{24a}$$

where

$$M^{sj} = \begin{bmatrix} M^s_{ii} & M^s_{iib} & 0 & 0 & 0 \\ M^s_{bii} & M^s_{ib} & M^s_{ibb} & 0 & 0 \\ 0 & M^s_{bib} & M^s_{bb} + M^j_{bb} & M^j_{bib} & 0 \\ 0 & 0 & M^j_{ibb} & M^s_{ib} & M^j_{bii} \\ 0 & 0 & 0 & M^j_{iib} & M^j_{ii} \end{bmatrix}$$

$$K^{sj} = \begin{bmatrix} K^s_{ii} & K^s_{iib} & 0 & 0 & 0 \\ K^s_{bii} & K^s_{ib} & K^s_{ibb} & 0 & 0 \\ 0 & K^s_{bib} & K^s_{bb} + K^j_{bb} & K^j_{bib} & 0 \\ 0 & 0 & K^j_{ibb} & K^s_{ib} & K^j_{bii} \\ 0 & 0 & 0 & K^j_{iib} & K^j_{ii} \end{bmatrix}$$

$$x^{sj} = [x_{is}, x_{ibs}, x_{bsj}, x_{ibj}, x_{ij}]^T \tag{24b}$$

$$f^{sj} = [f^s_i, f^s_{ib}, f^{sj}_b, f^j_{ib}, f^j_i]^T \tag{24c}$$

Thus, using the isolated boundary loading to generate the component models, and this superposition-based synthesis process, the exact model of the coupled structure is reproduced. As compared with the subsystem decomposition approach, in which interconnections are discarded in local decoupled subsystem models [such as Eq. (5)], the developed component model captures the dynamic interactions with its neighboring components. It is, therefore, reasonable to expect that the component controllers designed with these component models would produce better decentralized control actions.

The component models developed using this new approach have a direct connection with the subsystem decomposition approach. These models are identical to the decoupled subsystem models, if an overlapping decomposition is applied to the high-fidelity model. This is a key connection, which allows the use of large-scale system analysis tools developed in Ikeda and Šiljak[17] and Ikeda et al.[18] The report by Šiljak[19] contains a more elaborate exposition of the development of overlapping decomposition for decentralized control, and the tools for evaluating the performance of the controlled coupled structure after the controlled component designs have been completed. In Young,[3,4] decentralized control for truss structures has been investigated using decoupled subsystem models that are obtained by an overlapping decomposition of the structure. The performance analysis carried out in those earlier works can essentially be applied herein.

Interlocking Control Concept

The new insights gained from the newly developed component modeling approach in turn motivate a new component level control design concept, which we call an interlocking control (ILC) concept. Again, we use the two-component structure to illustrate the idea. First, consider the model that results from applying overlapping decomposition to the high-fidelity model. It is an ISOS model, given by

$$\begin{bmatrix} M_{ii}^p & M_{iib}^p & 0 \\ M_{bii}^p & M_{ib}^p & M_{ibb}^p \\ 0 & M_{bib}^p & M_{bb,iL}^s \end{bmatrix} \begin{bmatrix} \ddot{x}_{ip} \\ \ddot{x}_{ibp} \\ \ddot{x}_{bp} \end{bmatrix} + \begin{bmatrix} K_{ii}^p & K_{iib}^p & 0 \\ K_{bii}^p & K_{ib}^p & K_{ibb}^p \\ 0 & K_{bib}^p & K_{bb,iL}^s \end{bmatrix} \begin{bmatrix} x_{ip} \\ x_{ibp} \\ x_{bp} \end{bmatrix}$$

$$= \begin{bmatrix} f_i^p \\ f_{ib}^p \\ f_b^p \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_{bib}^q \ddot{x}_{ibq} + K_{bib}^q x_{ibq} + f_b^q \end{bmatrix} \tag{25}$$

for all p and q in the index set $(p,q) = \{(s,j), (j,s)\}$. Note that the Eq. (25), with the second term on the right-hand side removed, is identical to a component model for CCS. ILC refers to a component level control design concept in which collocated actuators and sensors are placed at the internal boundary degree of freedom, and the control law is designed, using the developed component model for CCS, to minimize the internal boundary coordinate motion. It is motivated by a closer examination of the preceding equation, which shows that such minimization would localize the dynamic interactions of the coupled structure in the components. The component control action is designed to lock up its own internal boundary to realize a boundary condition that better approximates the one assumed in the component modeling of its adjacent components.

A convenient control design technique for this concept is the linear quadratic optimal regulator approach, in which the internal boundary coordinates are considered as regulated outputs of the component to be weighted together with the component control inputs in the quadratic performance index. The resulting component control law minimizes this index. Using this approach, the ILC concept translates into a two-step component control design process summarized as follows.

1) For the sth component, use the component model

$$\begin{bmatrix} M_{ii}^s & M_{iib}^s & 0 \\ M_{bii}^s & M_{ib}^s & M_{ibb}^s \\ 0 & M_{bib}^s & M_{bb,iL}^s \end{bmatrix} \begin{bmatrix} \ddot{x}_{is} \\ \ddot{x}_{ibs} \\ \ddot{x}_{bs} \end{bmatrix} + \begin{bmatrix} K_{ii}^s & K_{iib}^s & 0 \\ K_{bii}^s & K_{ib}^s & K_{ibb}^s \\ 0 & K_{bib}^s & K_{bb,iL}^s \end{bmatrix} \begin{bmatrix} x_{is} \\ x_{ibs} \\ x_{bs} \end{bmatrix} = \begin{bmatrix} 0 \\ u^s \\ 0 \end{bmatrix}$$

$$y^s = x_{ibs} \tag{26}$$

for control system design, where u^s and y^s denotes, respectively, the control force exerted by the actuators, and the sensor outputs, at the internal boundary coordinates.

2) Derive the component control law by minimizing the performance index

$$J_c^s = \frac{1}{2} \int_0^\infty (y^{sT} y^s + u^{sT} R^s u^s)\, dt \tag{27}$$

The resulting optimal component control is generally a state feedback control law

$$u^s = [G_{di}^s \quad G_{dibs}^s \quad G_{db}^s] \begin{bmatrix} x_{is} \\ x_{ibs} \\ x_{bs} \end{bmatrix} + [G_{ri}^s \quad G_{ribs}^s \quad G_{rb}^s] \begin{bmatrix} \dot{x}_{is} \\ \dot{x}_{ibs} \\ \dot{x}_{bs} \end{bmatrix} \tag{28}$$

Whereas the linear quadratic optimal regulator approach may require full component state feedback, other control design techniques that produce component controllers that use static and dynamic output feedback can also be used in the ILC concept, provided the internal boundary coordinate motion is minimized.

At first glance, the component control law is a decentralized control law, since it feeds back only measurements of the component. If we consider a state space whose state vector is composed of the components' coordinates and their time derivatives in the coupled structure, the component control law is indeed a decentralized one in this space. However, the state space of the coupled structure is a contraction[17] of this space, that is, it is the state space with all the duplicated boundary coordinates and their time derivatives removed. For the two-component structure, whose model is given by Eq. (24), the component control law given by Eq. (28) can be expressed in the contracted state space as

$$\begin{bmatrix} u^s \\ u^j \end{bmatrix} = \begin{bmatrix} G_{di}^s & G_{dibs}^s & G_{db}^s & 0 & 0 \\ 0 & 0 & G_{db}^j & G_{dibj}^j & G_{dij}^j \end{bmatrix} x^{sj}$$

$$+ \begin{bmatrix} G_{ri}^s & G_{ribs}^s & G_{rb}^s & 0 & 0 \\ 0 & 0 & G_{rb}^j & G_{ribj}^j & G_{rij}^j \end{bmatrix} \dot{x}^{sj} \tag{29}$$

showing that the boundary states are fed back to the adjacent component controllers.

Applications to Truss Structure Control

The developed CCS method is applied to the design of structural control laws for a planar truss structure for a preliminary assessment of its feasibility toward solving more complex structural control design problems. This truss structure, which is depicted in Fig. 3, has six bays, and the nodal coordinates are defined as the vertical and horizontal displacements at the joints. The internal boundary degree of freedom at which collocated force actuators and displacement sensors are placed are marked by Δ.

External forces applied at the nodes are decomposed into orthogonal components. The assumptions made are that the truss members are subjected to axial forces alone, and not bending moments. The members are uniform rods of identical lengths L, mass per unit length m, cross-sectional area per unit length A, and modulus of elasticity E.

The six-bay truss can be viewed as a structure consisting of three identical components, namely, the left component, the center component, and the right component, which are com-

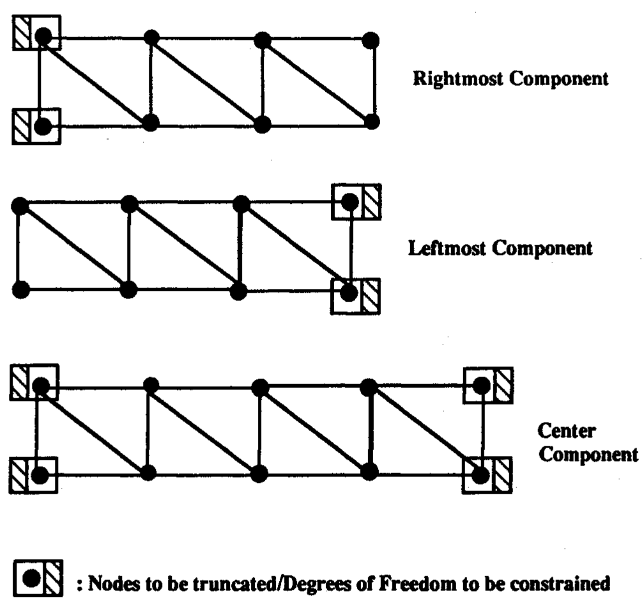

Fig. 3 The three expanded components.

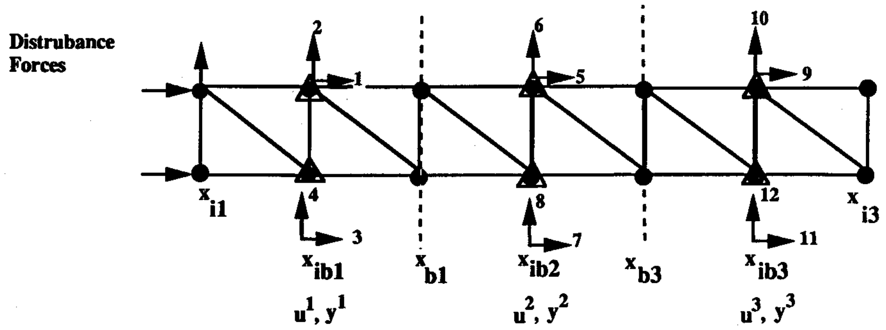

Fig. 4 Planar truss for CCS evaluation.

1.3536	-0.3536	0.0000	0.0000	-1.0000	0.0000	-0.3536	0.3536	0.0000	0.0000	0.0000	0.0000
-0.3536	1.3536	0.0000	-1.0000	0.0000	0.0000	0.3536	-0.3536	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	-1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-1.0000	0.0000	0.0000	0.0000	2.3536	-0.3536	0.0000	0.0000	-1.0000	0.0000	-0.3536	0.3536
0.0000	0.0000	0.0000	0.0000	-0.3536	1.3536	0.0000	-1.0000	0.0000	0.0000	0.3536	-0.3536
-0.3536	0.3536	-1.0000	0.0000	0.0000	0.0000	2.3536	-0.3536	0.0000	0.0000	-1.0000	0.0000
0.3536	-0.3536	0.0000	0.0000	0.0000	-1.0000	-0.3536	1.3536	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	2.3536	-0.3536	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.3536	1.3536	0.0000	-1.0000
0.0000	0.0000	0.0000	0.0000	-0.3536	0.3536	-1.0000	0.0000	0.0000	0.0000	2.3536	-0.3536
0.0000	0.0000	0.0000	0.0000	0.3536	-0.3536	0.0000	0.0000	0.0000	-1.0000	-0.3536	1.3536

a) Stiffness matrix of end component

2.3536	-0.3536	0.0000	0.0000	-1.0000	0.0000	-0.3536	0.3536	0.0000	0.0000	0.0000	0.0000
-0.3536	1.3536	0.0000	-1.0000	0.0000	0.0000	0.3536	-0.3536	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	2.3536	-0.3536	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	-1.0000	-0.3536	1.3536	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-1.0000	0.0000	0.0000	0.0000	2.3536	-0.3536	0.0000	0.0000	-1.0000	0.0000	-0.3536	0.3536
0.0000	0.0000	0.0000	0.0000	-0.3536	1.3536	0.0000	-1.0000	0.0000	0.0000	0.3536	-0.3536
-0.3536	0.3536	-1.0000	0.0000	0.0000	0.0000	2.3536	-0.3536	0.0000	0.0000	-1.0000	0.0000
0.3536	-0.3536	0.0000	0.0000	0.0000	-1.0000	-0.3536	1.3536	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	2.3536	-0.3536	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.3536	1.3536	0.0000	-1.0000
0.0000	0.0000	0.0000	0.0000	-0.3536	0.3536	-1.0000	0.0000	0.0000	0.0000	2.3536	-0.3536
0.0000	0.0000	0.0000	0.0000	0.3536	-0.3536	0.0000	0.0000	0.0000	-1.0000	-0.3536	1.3536

b) Stiffness matrix of center component

Fig. 5 Component stiffness matrix.

posed of the leftmost, the middle, and the rightmost two bays, respectively. The six-bay/three-component truss structure is chosen to capture the essential characteristics of a truss consisting of an arbitrary number of identical components, i.e., a truss structure with an arbitrarily large number of two-bay components is composed of the three types of components identified in the six-bay truss, with the center component duplicated as necessary. Thus, conclusions from the six-bay/three-component design apply equally well to the design of structural controls for a multiple bay truss.

CCS Modeling

For CCS, the component models are developed using the expanded component introduced in isolated boundary loading. The mass and stiffness matrices of the expanded component are derived, using a finite-element method with the Ritz-Rayleigh approximation. The truss member mass and stiffness matrices expressed with respect to local coordinates and used in the assembly process are[20]

$$K_{\text{member}} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \qquad M_{\text{member}} = \frac{mL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{30}$$

The component model is further scaled to remove the effects of the material properties. A new time variable $\tau = (m/6EA)^{1/2} Lt$ is introduced, and the nodal forces are scaled by L/EA.

The three expanded components from which the component models are derived are shown in Fig. 4. The rows and columns of the expanded components mass and stiffness matrices corresponding to the internal boundary nodes of the adjacent components are deleted to form the component's mass and stiffness matrices. These nodes are also identified in Fig. 4. The mass and stiffness matrices of the end and center components are given in Figs. 5 and 6. Due to symmetry, the matrices for the leftmost and the rightmost components are the same after permutation. Each of the stiffness and mass matrices is partitioned into 4×4 submatrices, and the submatrices corresponding to the boundary are highlighted. Figure 7 illustrates how the coupled structure model can be synthesized from the component models. The labels *i*, *ib*, and *b*, respectively, denote the submatrices of the internal, internal boundary, and boundary coordinates.

No damping is assumed for the truss structure, thus the open-loop poles of the coupled structure, and the three components, all lie on the imaginary axis. In Fig. 8, the magnitudes of the imaginary parts of these poles are plotted against the mode numbers. Note that since the coupled structure is a planar truss and is unconstrained, there are three rigid body modes—the first three modes in this plot corresponding to the triple double poles at the origin of the complex plane. The absence of rigid body modes in the component models, as indicated by the nonzero magnitudes of their poles, results

3.4142	-1.4142	0.0000	0.0000	1.0000	0.0000	0.7071	-0.7071	0.0000	0.0000	0.0000	0.0000
-1.4142	3.4142	0.0000	1.0000	0.0000	0.0000	-0.7071	0.7071	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	2.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	2.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.0000	0.0000	0.0000	0.0000	5.4142	-1.4142	0.0000	0.0000	1.0000	0.0000	0.7071	-0.7071
0.0000	0.0000	0.0000	0.0000	-1.4142	3.4142	0.0000	1.0000	0.0000	0.0000	-0.7071	0.7071
0.7071	-0.7071	1.0000	0.0000	0.0000	0.0000	5.4142	-1.4142	0.0000	0.0000	1.0000	0.0000
-0.7071	0.7071	0.0000	0.0000	0.0000	1.0000	-1.4142	3.4142	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	5.4142	-1.4142	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.4142	3.4142	0.0000	1.0000
0.0000	0.0000	0.0000	0.0000	0.7071	-0.7071	1.0000	0.0000	0.0000	0.0000	5.4142	-1.4142
0.0000	0.0000	0.0000	0.0000	-0.7071	0.7071	0.0000	0.0000	0.0000	1.0000	-1.4142	3.4142

a) Mass matrix of end component

5.4142	-1.4142	0.0000	0.0000	1.0000	0.0000	0.7071	-0.7071	0.0000	0.0000	0.0000	0.0000
-1.4142	3.4142	0.0000	1.0000	0.0000	0.0000	-0.7071	0.7071	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	5.4142	-1.4142	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	-1.4142	3.4142	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.0000	0.0000	0.0000	0.0000	5.4142	-1.4142	0.0000	0.0000	1.0000	0.0000	0.7071	-0.7071
0.0000	0.0000	0.0000	0.0000	-1.4142	3.4142	0.0000	1.0000	0.0000	0.0000	-0.7071	0.7071
0.7071	-0.7071	1.0000	0.0000	0.0000	0.0000	5.4142	-1.4142	0.0000	0.0000	1.0000	0.0000
-0.7071	0.7071	0.0000	0.0000	0.0000	1.0000	-1.4142	3.4142	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	5.4142	-1.4142	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-1.4142	3.4142	0.0000	1.0000
0.0000	0.0000	0.0000	0.0000	0.7071	-0.7071	1.0000	0.0000	0.0000	0.0000	5.4142	-1.4142
0.0000	0.0000	0.0000	0.0000	-0.7071	0.7071	0.0000	0.0000	0.0000	1.0000	-1.4142	3.4142

b) Mass matrix of center component

Fig. 6 Component mass matrix.

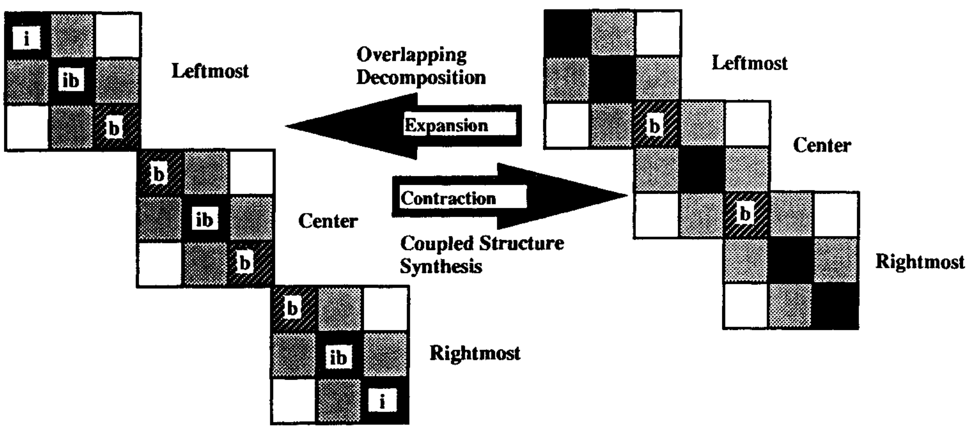

Fig. 7 Synthesis of six-bay truss model from component models.

from the boundary constraints imposed on the expanded components.

Interlocking Control Design

A linear quadratic regulator approach to ILC, as outlined in the last section, is adopted in the design of the component controllers. An identical control weighting matrix $R = 0.001\, I_{4\times 4}$ for all three components is chosen. The control designs for the leftmost and rightmost components are identical due to symmetry; therefore, we only need to carry out a center component design and an end component control design. In each of these component control designs, a 24th-order Riccati equation is solved to compute the optimal feedback gains, in contrast to a 56th-order Riccati equation, if an optimal centralized control approach is used. However, for an N bay truss, the computational burden of the component control design remains to be a 24th-order Riccati equation, whereas in the centralized case, the burden is an $8\,(N+1)$-order Riccati equation. The resulting state feedback gain matrices for the two control designs are given in Fig. 9. The gain matrices for the leftmost and rightmost component are identical, after

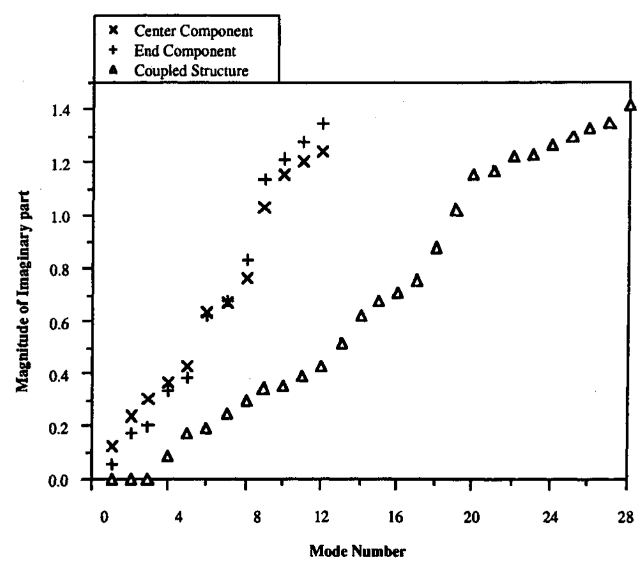

Fig. 8 Open-loop poles of the component and the coupled structure.

i				ib				b			
-0.9441	-0.0533	-0.0051	0.0611	-29.3434	-0.3830	0.0307	-0.1078	-0.9508	-0.1098	-0.3905	0.4483
-0.0650	-0.0069	0.0065	0.0104	-0.2615	-30.2898	0.0683	-0.9748	-0.0604	0.1043	0.4307	-0.4461
-0.3922	0.4230	-0.9634	-0.0764	0.0399	-0.1056	-29.3384	-0.3757	-0.0024	0.0314	-0.9574	-0.0197
0.4272	-0.4638	-0.0680	0.1004	0.0325	-0.9652	-0.2332	-30.2908	0.0075	0.0025	-0.0657	-0.0034

a) Position feedback gain matrix for end component

i				ib				b			
-2.0169	-0.3745	-0.0032	-0.4141	-17.5441	2.5549	0.0359	-0.2564	-2.0506	0.1624	-0.9461	1.0004
-0.0272	-0.0071	0.0078	0.0024	2.6298	-13.8370	-0.1726	-2.5240	0.0094	-0.2311	0.9939	-1.1863
-0.9958	1.3694	-1.9783	0.3249	0.0614	-0.2249	-17.5521	2.6047	0.0190	-0.1692	-2.0147	-0.0779
1.0196	-1.3908	-0.0308	-0.3135	-0.2426	-2.5193	2.6775	-13.8328	0.0084	-0.0002	-0.0273	-0.0039

b) Velocity feedback gain matrix for end component

i				ib				b			
-0.9422	-0.0195	-0.0017	0.0312	-29.3318	-0.3777	0.0131	-0.1031	-0.9417	-0.1090	-0.3860	0.4439
-0.0667	-0.0025	0.0061	0.0017	-0.2901	-30.2897	0.0648	-0.9679	-0.0595	0.1047	0.4309	-0.4466
-0.3860	0.4439	-0.9417	-0.1090	0.0131	-0.1031	-29.3318	-0.3777	-0.0017	0.0312	-0.9422	-0.0195
0.4309	-0.4466	-0.0595	0.1047	0.0648	-0.9679	-0.2901	-30.2897	0.0061	0.0017	-0.0667	-0.0025

c) Position feedback gain matrix for center component

i				ib				b			
-2.0062	-0.0777	0.0195	-0.1689	-17.5242	2.5323	-0.0004	-0.2236	-2.0456	0.1624	-0.9438	0.9980
-0.0292	-0.0038	0.0079	-0.0007	2.5927	-13.8438	-0.1761	-2.5063	0.0089	-0.2310	0.9935	-1.1861
-0.9438	0.9980	-2.0456	0.1624	-0.0004	-0.2236	-17.5242	2.5323	0.0195	-0.1689	-2.0062	-0.0777
0.9935	-1.1861	0.0089	-0.2310	-0.1761	-2.5063	2.5927	-13.8438	0.0079	-0.0007	-0.0292	-0.0038

d) Velocity feedback gain matrix for center component

Fig. 9 Linear quadratic component controller gains.

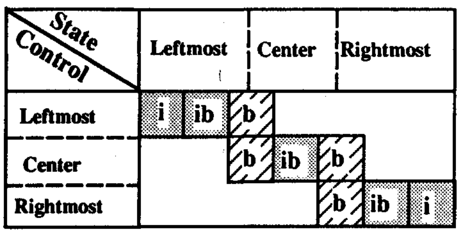

Fig. 10 Feedback matrix form for six-bay truss.

permutation. These gain matrices are also partitioned into 4×4 submatrices, which are identified with the internal, internal boundary, and boundary coordinates by *i*, *ib*, and *b*, respectively. Figure 10 illustrates how the feedback gain matrix for the six-bay truss is synthesized from the gain matrices in Fig. 9. The nonzero blocks of this matrix are highlighted.

The controlled components' poles, as well as the poles of the controlled truss structure, are plotted in Figs. 11 and 12. Since the leftmost and rightmost components are identical, we plot only the poles of one of them, which are denoted by end component poles in these figures. All the poles of the controlled structure have negative real parts, indicating that the closed-loop system is asymptotically stable. That the pole locations of the controlled components are close to that of the controlled coupled structure indicates that the component models developed for CCS are effective for this structural control design.

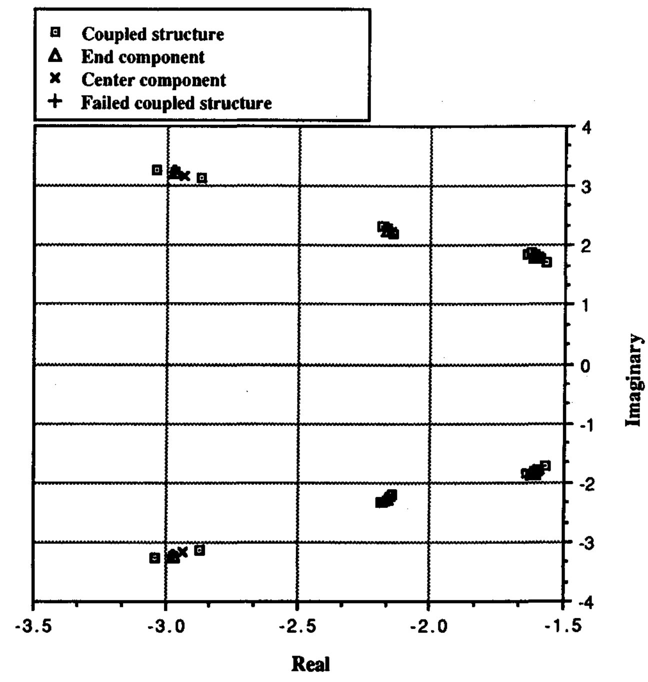

Fig. 11 Pole locations of the controlled components and the controlled truss—large real parts.

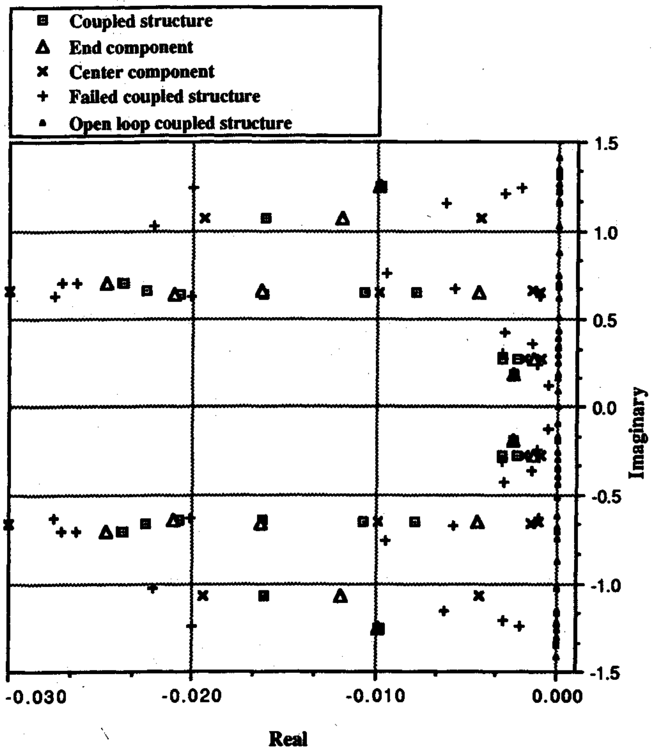

Fig. 12 Pole locations of the controlled components and the controlled truss—small real parts.

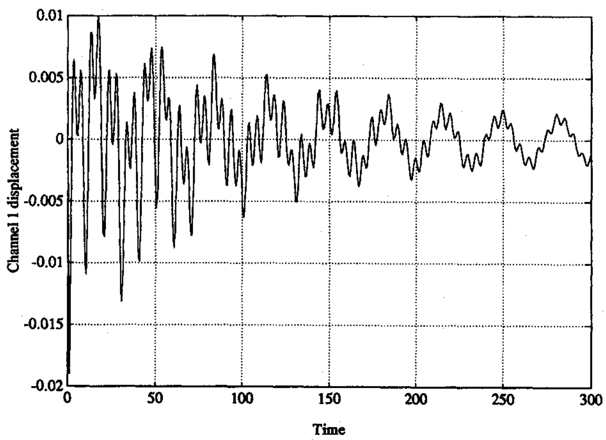

Fig. 13a Channel 1 displacement time response.

For transient response studies, we examine the response of the controlled structure to three disturbance force pulses—unity amplitude, 0.5 pulse width—simultaneously applied to the leftmost nodes of the truss, as shown in Fig. 4. The coupled structure is assumed to be in static equilibrium initially in the simulation. A sample of the sensor output time responses are shown in Figs. 13a–13d. Four of the 12 sensor channels, two horizontal (channels 1 and 5 in Fig. 4) and two vertical nodal displacements (channels 8 and 10) at the internal boundaries of the controlled components, are selected. The magnitudes of the displacement response drop by an order of magnitude per component for nodes that are further away from the disturbances. The delay effect of the force pulses on the displacements shown in Fig. 13d is typical for the right-

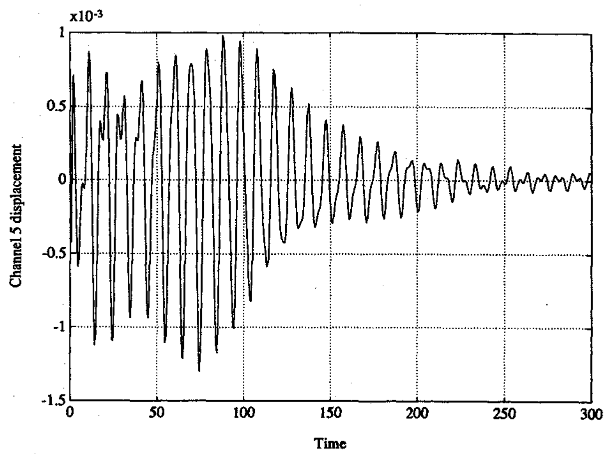

Fig. 13b Channel 5 displacement time response.

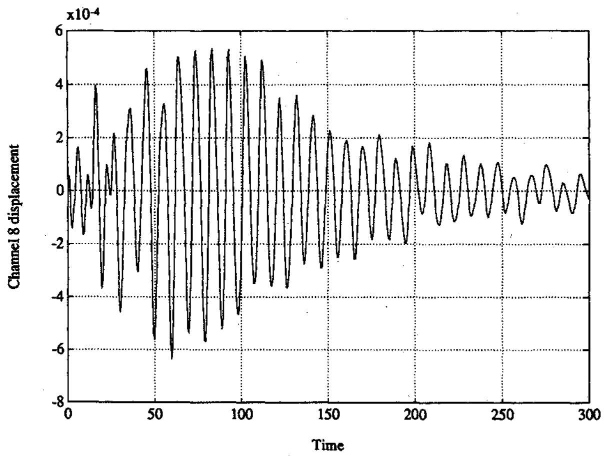

Fig. 13c Channel 8 displacement time response.

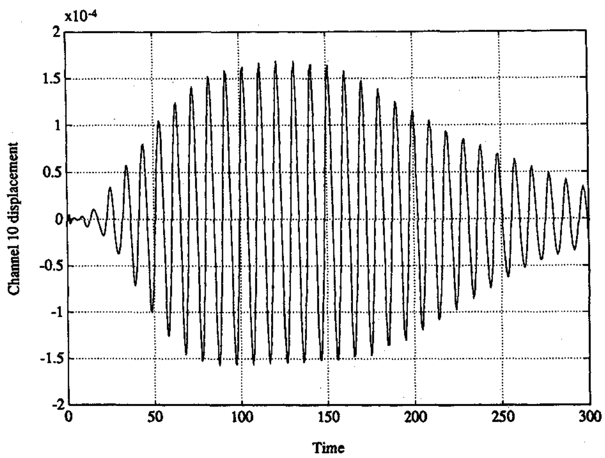

Fig. 13d Channel 10 displacement time response.

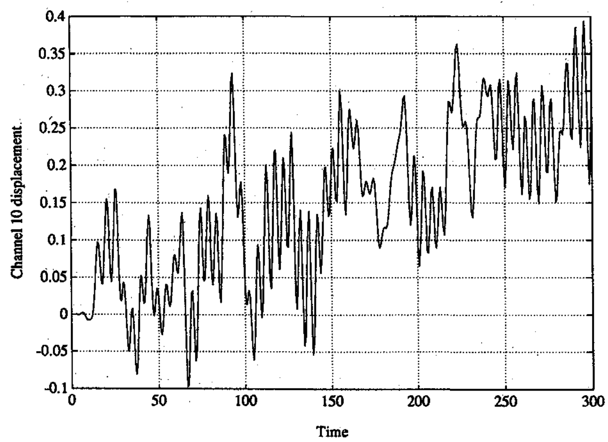

Fig. 14 Channel 10 displacement time response, uncontrolled.

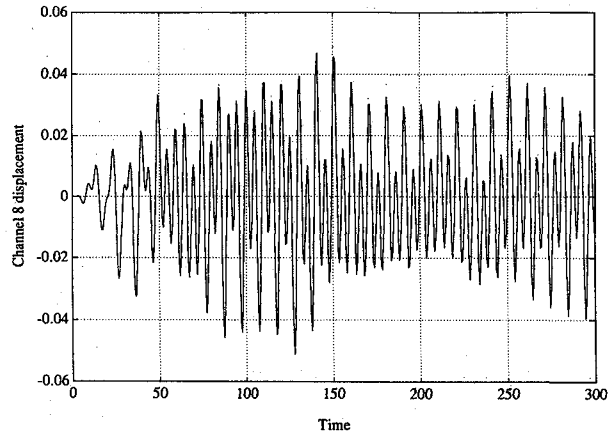

Fig. 15c Channel 8 displacement time response, with center component controller failure.

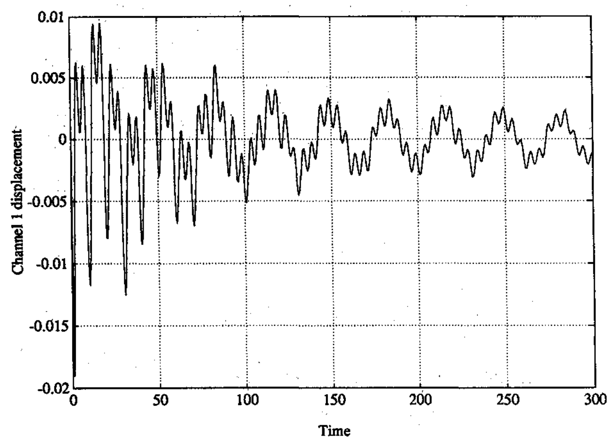

Fig. 15a Channel 1 displacement time response, with center component controller failure.

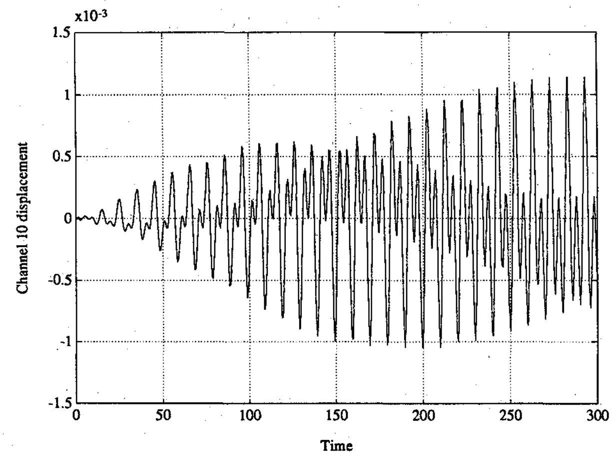

Fig. 15d Channel 10 displacement time response, with center component controller failure.

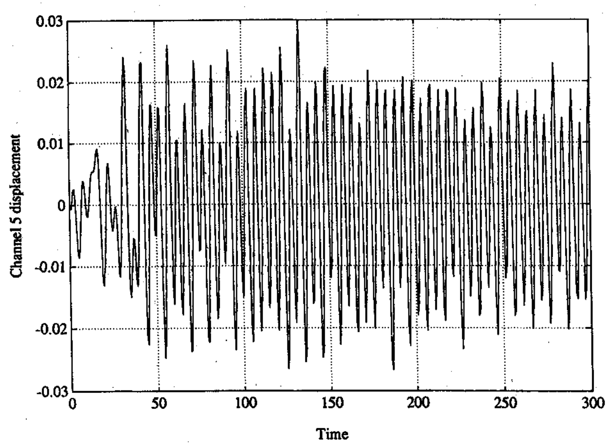

Fig. 15b Channel 5 displacement time response, with center component controller failure.

most component. That the controlled components stabilize the rigid body modes of the structure is illustrated with Fig. 14, which shows the response of channel 10 for the uncontrolled structure.

Controller Failure

That the developed CCS method inherits the capability to withstand system failures from decentralized control developed using the subsystem decomposition approach is demonstrated with the following results for the controlled structure, in which the center component controller failed. Figures 11 and 12, in which the pole locations of the controlled structure with the component controller failure are also plotted, show a slight decrease in damping in a number of the modes near the imaginary axis. The controlled structure, however, remains to be asymptotically stable. This failure scenario is simulated for the same disturbances and initial conditions as before. The same four channel responses are included herein as Figs. 15a–15d. Figures 15b and 15c show an order of magnitude performance degradations for the two displacements at the center component – from the $O(10^{-3})$ peak displacement excursion for channel 5 and $O(10^{-4})$ for channel 8 in the no-failure state, to $O(10^{-2})$ and $O(10^{-2})$, respectively, in the failure state. However, despite the center component controller failure, the neighboring components stabilize the vibrations in the center component with interlocking controls.

Conclusions

The controlled component synthesis method has been shown in this paper to be an effective tool for the design of distributed control for truss structures. Although the distributed finite-element modeling phase in this method is similar in many aspects to component mode synthesis, which has been a widely accepted tool in the analysis of complex structures, our conclusion, upon a critical evaluation of its suitability for distributed structural control, is that new methods for developing component models and approximate models of complex structures are needed. In this paper, a new component modeling approach that is more appropriate for controlled component synthesis is proposed. We also showed that the model of a large space structure is perhaps too large to be engaged in existing decentralized control methods, particularly those using the information constraint approach. The model dimension of the planar truss considered herein rapidly increases as the number of bays gets large—a 100-bay truss, similar to the one in Fig. 4, means 808 states. The proposed distributed control method effectively attacks the spillover problem by dealing with a higher fidelity model, while keeping the computational problem associated with the control design manageable.

The modeling and design of distributed structural controllers for a planar truss demonstrate the feasibility of controlled component synthesis for structures with an open chain topology. Ring trusses and other similar closed chain structures, however, can also be handled by this approach using an assembly process which generates the controlled closed chain structure from a hierarchy of open chain supercomponents. Continued research in extending the proposed method has been in the directions of examining the robustness of the distributed controllers to component modeling errors, pursuing alternative component control designs using the interlocking control concept, and developing a hierarchical controlled component approach for complex space structures.

Acknowledgment

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